Data-aware algorithms for matrix and tensor computations

Suraj Kumar

Inria & ENS Lyon

Email:*suraj.kumar@ens-lyon.fr*

CR12: September 2024

https://surakuma.github.io/courses/daamtc.html

Table of Contents

Course details

Topics:

- Communication costs on sequential and parallel machines
- Matrix computations
- Tensor computations
- Popular ways to work with large matrix and tensor data

Class hours: Tuesday 08:00 AM (room A1)
+ Thursday 03:45 PM (room B1)

Contact: suraj.kumar@ens-lyon.fr/suraj.kumar@inria.fr

Course details

Evaluation:

- 4 homework assignments (best 3 would be counted) (40%)
- Project (60%): 1/2 students, paper(s) reading + report + presentation

Website:

- `https://surakuma.github.io/courses/daamtc.html`
- Course material available online (right after class)
- Link to assignments and other resources

General stuff:

- Stop me with questions, ask for re-explanation
- Will have catch-up sessions after each 2 weeks
- Feel free to email your doubts

Table of Contents

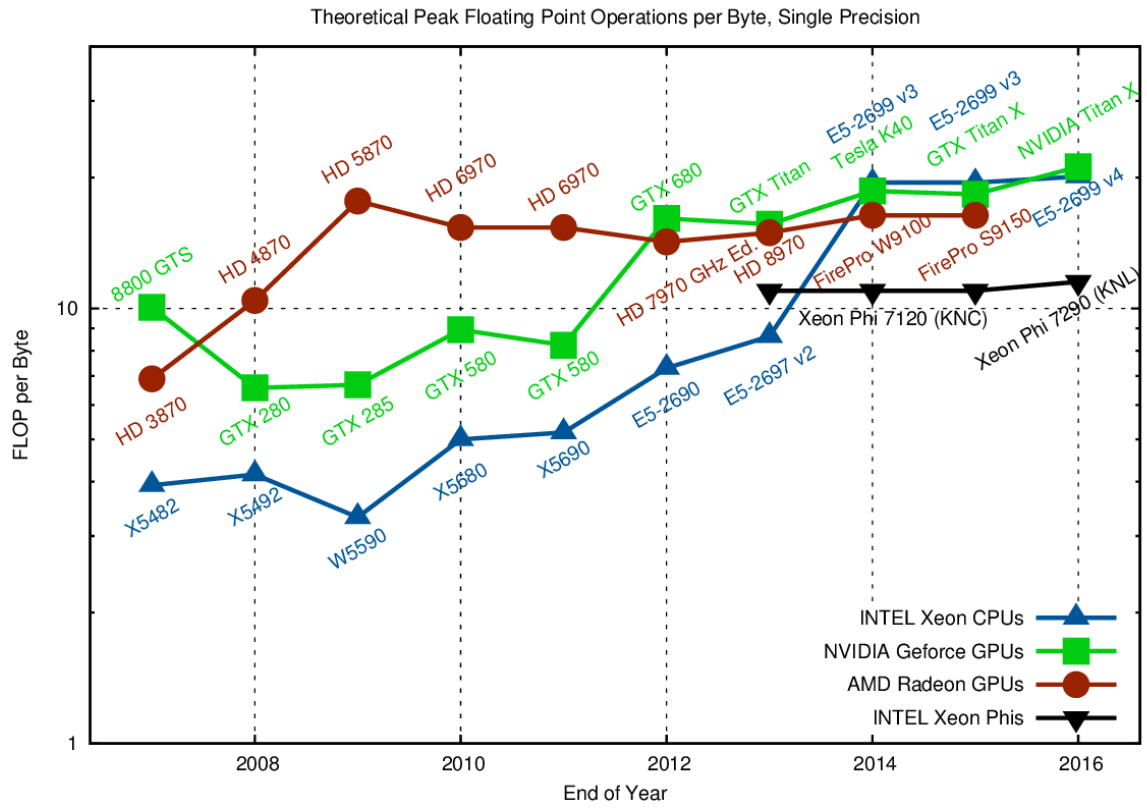
Motivation

- (Fast) Memory: place to store data to compute
- Always been a limited resource (4KB in Appollo 11 computer)
- 64KB L1 cache in recent iphone
- Problem size is always getting bigger ...
- Annual improvements:
  - Time per flop (computation): 59%
  - Data movement (communication):

	Bandwidth	Latency
Network	26%	15%
DRAM	23%	5%

Data from *Getting up to speed: The future of supercomputing*, 2005, National Academies Press (2004 figure based on data on the period 1988-2002)

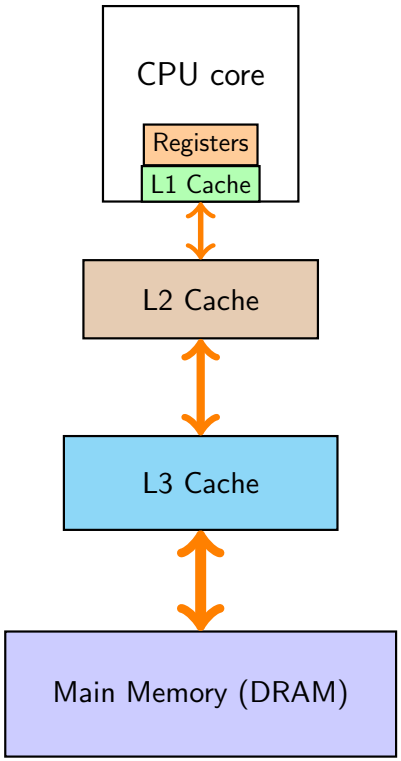
Flop per byte moved ratio



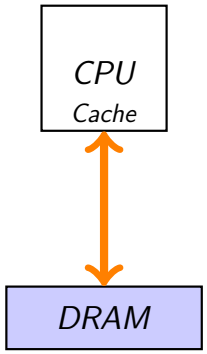
From `http://www.karlrupp.net/2013/06/cpu-gpu-and-mic-hardware-characteristics-over-time/`

- Number of flops performed in the time needed to move a byte
- $\frac{\text{computing speed}}{\text{communication speed}}$
- Avoid communication to save time (and energy)

Simplified model for a sequential CPU core

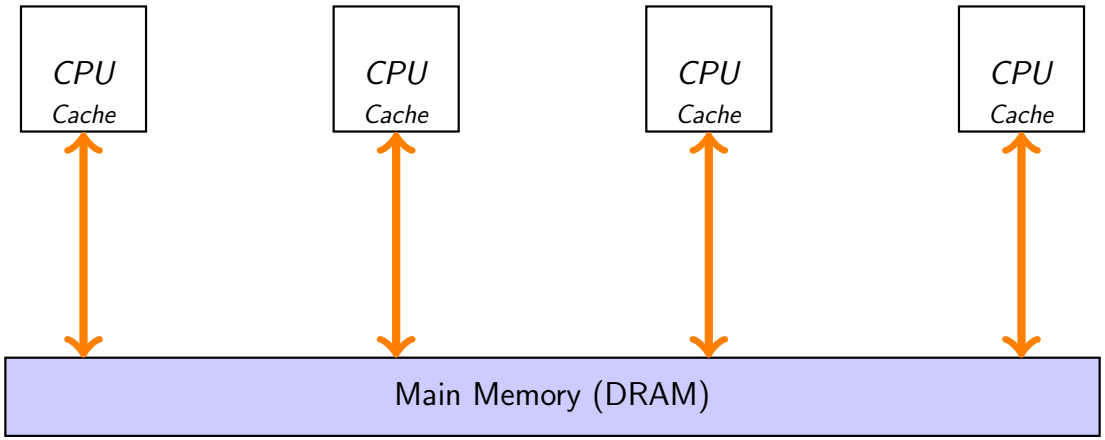


Cache hierarchy model



Simplified model

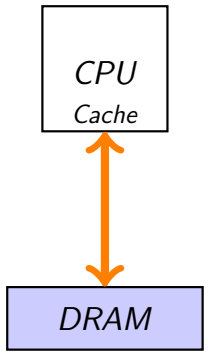
Shared memory machine model



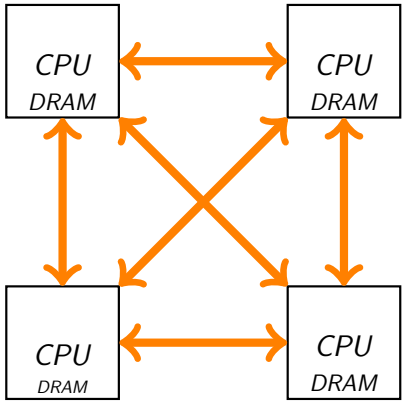
Shared memory parallel machine model

- We can view all CPUs as a single large processing unit

Data transfer models in this course



Sequential machine

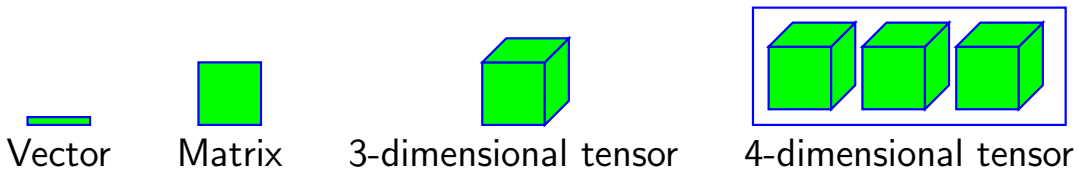


Distributed memory homogeneous machine

- α: per-message latency cost, β: per-word bandwidth cost, γ: per-operation arithmetic cost
- An algorithm's total cost $= \alpha \cdot$ (#messages) $+ \beta \cdot$(#data transfers) $+ \gamma \cdot$ (#computations)
- For a parallel machine, we consider cost along the critical path
- Concentrate mainly on bandwidth costs throughout the course

Sometimes computations can be overlapped with data transfers. However to keep the models simple we will not take that into account throughout the course.

Tensors and their uses



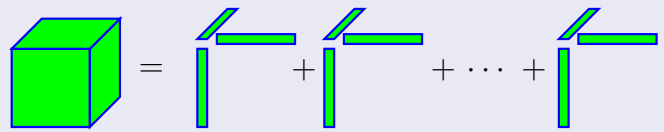
- **Neuroscience**: Neuron $\times$ Time $\times$ Trial
- **Media**: User x Movie x Time
- **Ecommerce**: User x Product x Time
- **Social-Network**: Person x Person x Time x Type

Tensors and their uses

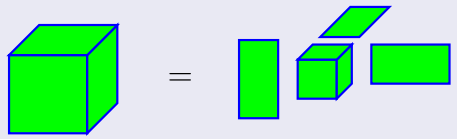
- High dimensional tensors: Neural network, Molecular simulation, Quantum computing

- People work with low dimensional structure (decomposition) of the tensors

Canonical decomposition



Tucker decomposition



Tensor-train decomposition